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NONLINEAR DYNAMICS IN QUANTUM PHYSICS–THE STUDY OF QUANTUM CHAOS

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Abstract:
We use the quantum action to study the dynamics of quantum system at finite temperature. We construct the quantum action non-perturbatively and find temperature dependent action parameters. Here we apply the quantum action to study quantum chaos. We present a numerical study of a classically chaotic 2-D Hamiltonian system - harmonic oscillators with anharmonic coupling. We compare Poincaré sections for the quantum action at finite temperature with those of classical action.

Keywords. Quantum propogator, quantum action, quantum chaos, flux equation, Poincaré section.

1 Introduction

Chaotic phenomena are found in microscopic physics ruled by quantum mechanics – an area presently under very active investigation. For reviews see Refs.[1, 12, 13, 24, 27]. Examples are the hydrogen atom in a strong magnetic field [11] and the quantum mechanical stadium billiard [23].

Classical chaos theory can not simply be taken over to quantum physics (due to Heisenberg's uncertainty relation). Hence workers have tried to characterize quantum chaos in ways alternative to classical chaos. The following approaches to quantum chaos have been considered:

(i) *Gutzwiller's trace formula.* Gutzwiller [12] has established a relation between the density of states of the quantum system and a sum over classical periodic orbits (periodic orbit quantisation). The trace formula has been applied in the semi-classical regime (e.g. highly excited states of atom). Wintgen [28] applied it to the diamagnetic hydrogen system and extracted periodic orbit information from experimental level densities.

(ii) *Effective action.* Another possibility is to use the conventional effective action [7, 18]. An effective action exists also at finite temperature