

PERMANENCE OF 2-HOST 1-PARASITOID SYSTEMS

Ryusuke Kon¹ and Yasuhiro Takeuchi¹

¹Department of Systems Engineering, Faculty of Engineering
Shizuoka University, Johoku 3-5-1, Hamamatsu, Shizuoka 432-8561, Japan

Abstract. In this paper, permanence of a host-parasitoid system, which is composed of two hosts and one parasitoid, is considered. Sufficient conditions for permanence of the system are obtained by using average Liapunov functions. One of the sufficient conditions shows that even if two hosts cannot coexist by themselves, they can coexist with a help of a parasitoid in a sense of permanence .

Keywords. permanence, host-parasitoid systems, average Liapunov functions, coexistence, dominance

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1 Introduction

In population ecology, it is important to consider coexistence of species. Concerning with an interaction between hosts and parasitoids, which is a popular interaction in insect populations (Hassell [4], [5]), Comins and Hassell [2] considered the following host-parasitoid model:

$$\begin{cases} H_1(t+1) &= \lambda_1 H_1(t) \exp[-\sum_{j=1}^2 \mu_{1j} H_j(t)] \exp[-a_1 P(t)] \\ H_2(t+1) &= \lambda_2 H_2(t) \exp[-\sum_{j=1}^2 \mu_{2j} H_j(t)] \exp[-a_2 P(t)] \\ P(t+1) &= \sum_{j=1}^2 b_j H_j(t) (1 - \exp[-a_j P(t)]), \end{cases} \quad (1)$$

$$(H_1(t), H_2(t), P(t)) \in \mathbf{R}_+^3 := \{(H_1, H_2, P) \in \mathbf{R}^3 : H_1 \geq 0, H_2 \geq 0, P \geq 0\}$$
$$t \in \mathbf{Z}_+ := \{0, 1, 2, \dots\},$$

where $\lambda_i > 1, \mu_{ij} > 0, a_i > 0, b_i > 0$ ($i, j = 1, 2$). This system is composed of two hosts, H_1 and H_2 , and one parasitoid, P , whose population densities at generation t are denoted by $H_1(t)$, $H_2(t)$ and $P(t)$, respectively. By investigating stability of a positive equilibrium point of system (1), Comins and Hassell [2] considered whether two hosts competing with each other can coexist with a help of the parasitoid. Since the species in discrete-time systems often coexist without a stable equilibrium point, in this paper we consider permanence (see Definition 1), which ensures coexistence of species irrespective of population dynamics. To emphasize an influence of a parasitoid on the possibility of coexistence of the competing hosts, we assume that one