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DISEASE MORTALITY IN EPIDEMIC MODELS

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Abstract. We consider models for the transmission of infectious diseases with an arbitrary infective period distribution in which there may be both disease fatalities and recoveries. The possibility of disease deaths complicates the analysis because the total population size can not be asymptotically constant. If the birth rate is constant there is always a unique asymptotically stable equilibrium. However, if the birth rate depends on susceptible population size, the endemic equilibrium may be unstable for some infective period distributions and the stability may depend on the fraction of infectives who recover from the disease.
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1 Introduction

In simple models for the transmission of communicable diseases which incorporate births and deaths as well as epidemiological effects it is often assumed that the total population size is constant, with a constant birth rate and a death rate in each class which is proportional to the size of the class [9,10]. This assumption can easily be generalized to allow a birth rate which is a function of total population size, provided there are no disease deaths. If there are no disease deaths the total population size approaches a limit and the theory of asymptotically autonomous systems [5,12] may be applied to treat the total population size as if it were constant.

If there are deaths caused by disease it is not possible to keep the total population size constant and the system can not be asymptotically autonomous. We will examine models, both of SIS type and of SIR type but concentrating on SIS models, in which there may be deaths due to disease. Initially, we will assume a constant birth rate. Although this assumption is unrealistic, it is the simplest way to include demographics in an epidemiological model but keep total population size bounded in the absence of disease.

A model with birth rate depending on susceptible population size is not asymptotically autonomous, even if there are no disease deaths. In a later section, we shall consider such models (which would be appropriate for severely