

## ON DUCKS IN THE MINIMAL SYSTEM WITH CODIMENSION TWO

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**Abstract.** In the modified Bonhoeffer-van der Pol system, its local model can not be induced well. By choosing an adequate regular transformation, which contains a new parameter  $u$ , it can be proved that this local model with codimension two becomes well induced. Then, the winding number for a duck solution tends to infinity as a regular limit for the new parameter. In case  $u - 1$  is fixed sufficiently small, the number becomes large as the value of a co-parameter  $b$  embedded originally tends to  $0_+$ , or  $-1_-$ .

**keywords.** modified Bonhoeffer-van der Pol system, constrained system, singular perturbation, duck solution, winding number

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### 1 Introduction

The modified Bonhoeffer-van der Pol (BVP) equations were proposed by H.Kawakami et al. [6] in 1999. Their results of a computer simulation for this system show that there exist winding orbits on some projected phase space. Furthermore, the winding number increases when some parameter contained originally in this system decreases.

The BVP equations are described as follows:

$$\begin{aligned}L_1 di_1/dt &= E_1 - R_1 i_1 - v, \\L_2 di_2/dt &= E_2 - R_2 i_2 - v, \\C dv/dt &= i_1 + i_2 + \rho(v),\end{aligned}\tag{1.1}$$

where  $i_1, i_2$  are the currents through the inductors  $L_1, L_2$  and the resistors  $R_1, R_2$ , respectively. Moreover,  $E_1, E_2$  are the constant voltages,  $v$  represents at the nonlinear resistor  $\rho$  ( $\rho(v) = v - v^3/3$ ) and  $C$  is a capacitor with very small capacitance.

Let us consider the following generalized BVP system:

$$\begin{aligned}dx/dt &= c_0 - a'x - az, \\dy/dt &= c_1 - b'y - bz, \\cdz/dt &= x + y + z - z^3/3,\end{aligned}\tag{1.2}$$