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AN ANALYTICAL SOLUTION TO THE STRUCTURAL DYNAMICS OF AN ELASTIC CYLINDER IN A CROSS FLOW

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Abstract. A theoretical analysis of the dynamic motion of an elastic structure vibrating freely in a flowing fluid has been carried out based on Hamilton principle. As an example, the free vibration of an elastic circular cylinder, fixed-supported at both ends, in a cross flow is investigated. The governing structural dynamic equations are solved assuming a model for the flow-induced forces, comprising a steady, a periodic and a random component. The effect of axial curvature of the cylinder resulting from the steady drag is considered for the first time. In order to solve the equations, parameters appearing in the force model, such as the Strouhal number, the root mean square (rms) lift and drag coefficient, and the fluid damping ratio, need to be specified. These parameters are determined from previous experiments. The analytical solution correctly predicts the occurrence of the first- and third-mode resonance, where the fluid excitation frequency is equal to the natural frequency of the fluid-cylinder system. The solution agrees reasonably well with experimental data in terms of the rms values of the vibration amplitude and the dynamic strain measured at the cylinder mid-span. It also gives a qualitative agreement with the measured spanwise distribution of the rms displacement at resonance and off resonance. When the reduced velocity, U_r, increases beyond 26.9, where the third-mode resonance in the lift direction occurs, analysis and measurement show a significant discrepancy in the vibration amplitude. This discrepancy could be attributed to nonlinear effects. Furthermore, axial curvature is found to have a significant effect on the system natural frequency and the vibration amplitude, especially in a water flow where the mean drag is large. The solution developed can be readily extended to cylinders of different cross-sections.
Keywords. Flow-induced vibration, Analytical solution, Fluid damping, Elastic cylinder.

1 INTRODUCTION

In a simple flow-induced vibration problem, where the structure is exposed to a uniform cross flow, the complexity depends to a very large extent on the properties of the structure. For example, if the structural stiffness is exceedingly large, the structure can be considered to be relatively rigid and fluid-structure interactions do not play a significant role. Of course, the structure