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MULTIPLE PURPOSE ALGORITHMS FOR INVARIANT MANIFOLDS

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Abstract. This paper deals with the numerical continuation of invariant manifolds, regardless of the restricted dynamics. Typically, invariant manifolds make up the skeleton of the dynamics of phase space. Examples include limit sets, co-dimension 1 manifolds separating basins of attraction (separatrices), stable/unstable/center manifolds, nested hierarchies of attracting manifolds in dissipative systems and manifolds in phase plus parameter space on which bifurcations occur. These manifolds are for the most part invisible to current numerical methods. The approach is based on the general principle of normal hyperbolicity, where the graph transform leads to the numerical algorithms. This gives a highly multiple purpose method. Examples of computations of both attracting and saddle-type (1D and 2D) manifolds will be given, with and without non-uniform adaptive refinement. A convergence result for the algorithm will be sketched.

Keywords. Invariant manifolds, normal hyperbolicity, chaotic dynamics, numerical continuation, bifurcation theory, computational geometry, graph transform.

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1 Introduction

Invariant manifolds of dynamical systems largely determine the geometry of their phase space. Codimension 1 manifolds, for example, may separate basins of attraction. But in general equilibria, closed curves, invariant tori, their stable and unstable manifolds etc. are the corner stones around which a more detailed analysis may be in order. If the invariant manifold of interest, say Σ_1, is no attractor, it may still lie in a higher dimensional invariant manifold, Σ_2, which is. If the dynamics are restricted to Σ_2 then Σ_1 may even serve as a separatrix. This is the kind of situation we want to look at in the present paper. To fix thoughts think of Σ_2 being a 3–torus attractor with phase lock dynamics, where the aim is to visualize an unstable 2–torus Σ_1 inside this 3–torus.

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