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BIFURCATION AND STABILITY ANALYSES OF A 13-D SEIC MODEL USING NORMAL FORM REDUCTION AND NUMERICAL SIMULATION [1]

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Abstract. Center manifold and normal form theories as well as numerical simulations are employed to analyse the bifurcation and stability of a 13-dimensional deterministic model associated with the transmission dynamics of two diseases within a host population. The stability of the multiple equilibrium solutions together with the associated possible critical points are discussed. It is shown that model can only undergo static bifurcations. Owing to the fact that the system exhibits a double-zero singularity, center manifold and normal form theories are used to reduce the model to a two-dimensional system. A detailed bifurcation and stability analysis is given for the behaviour of the system in the vicinity of the critical point. A robust finite-difference method is developed and used for the solution of the 13-D system. Unlike some conventional numerical schemes, this novel scheme does not suffer from scheme-dependent instabilities. The numerical method gives results that are consistent with the theoretical predictions.
Keywords. Equilibrium solution, center manifold, normal form, bifurcation and stability, finite-difference method.
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1 Introduction

Much of the classical work on modelling epidemiological systems has been restricted to the study of the dynamics between a host population and one infectious disease (see [1]). However, as noted by Rohani *et al.* [7], different diseases ecologically compete for the same finite group of susceptible hosts. This suggests that the population dynamics of one disease may be effected by the epidemics of another. This paper focuses on the bifurcation and stability

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