Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **10 (2003)** 301-316

GLOBAL STABILITY FOR A LOTKA-VOLTERRA COOPERATIVE OR COMPETITION SYSTEM WITH DISTRIBUTED DELAYS

Yasuhisa Saito[1]

[1] Department of Mathematical Sciences
Osaka Prefecture University, Sakai 599-8531, Japan

Abstract. This paper presents the necessary and sufficient condition for the global stability of the following Lotka-Volterra cooperative or competition system with distributed delays:

$$x'(t) = x(t)\left[r_1 - ax(t) + \alpha \int_{-\tau_{11}}^{0} x(t+\theta)d\mu_{11}(\theta) + \beta \int_{-\tau_{12}}^{0} y(t+\theta)d\mu_{12}(\theta)\right]$$
$$y'(t) = y(t)\left[r_2 - ay(t) + \beta \int_{-\tau_{21}}^{0} x(t+\theta)d\mu_{21}(\theta) + \alpha \int_{-\tau_{22}}^{0} y(t+\theta)d\mu_{22}(\theta)\right].$$

It is showed that the positive equilibrium of the system is globally asymptotically stable for all delays $\tau_{ij} > 0$ and measures $\mu_{ij}(\theta)$ with $\int_{-\tau_{ij}}^{0} d\mu_{ij}(\theta) = 1$ $(i, j = 1, 2)$, if and only if $|\beta| < a - \alpha$ and $|\beta| \leq a + \alpha$ hold.

Keywords. Lotka-Volterra, distributed delays, cooperative, competition, global asymptotic stability

AMS (MOS) subject classification: This is optional. But please supply them whenever possible.

1 Introduction

There have been many studies for the global stability of Lotka-Volterra systems with delays (see, [2-9] and the references cited therein). Most of the studies consider the situations in which undelayed intraspecific competitions are present. For example, Lu and Wang [5] obtained a necessary and sufficient condition for the global stability of a two-dimensional Lotka-Volterra system with undelayed intraspecific competitions but *without delayed intraspecific competitions*. The result was extended to an n-dimensional Lotka-Volterra system by Hofbauer and So [3]. However, we find few papers giving necessary and sufficient conditions for the global stability of Lotka-Volterra systems *with delayed intraspecific competitions*. Recently, in [6] and [8] the author obtained necessary and sufficient conditions for the global stability of Lotka-Volterra predator-prey, and cooperative or competition systems with delayed intraspecific competitions, respectively. Results in [6] have been generalized to a Lotka-Volterra predator-prey system with distributed delays (see [7]).