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ON SYNCHRONIZATION AND CROSS-TALK IN PARALLEL NETWORKS

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Abstract. We consider the effects of weak coupling between parallel copies of identical networks, themselves of arbitrary structural complexity. We work with additive networks with 'hard switching', *i.e.*, step function interactions. Two types of cross connections are studied, motivated by hypotheses concerning the role of cross-talk in synchronizing oscillations in otherwise independent parallel pathways in the motor circuitry involved in Parkinsonian tremor. Our results suggest that excitatory cross connections between corresponding units in parallel copies of a network promote synchronization when the individual networks are in a periodic regime, but not when they are chaotic. Forward projections within one copy of a network may stray to the corresponding receiving unit in a parallel network, and these cross connections also promote synchronization in the presence of a stable limit cycle when they have the same sign as the corresponding connection within one network, as long as this projection contributes to rather than retards the switching of the receiving unit on the stable limit cycle.
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1 Introduction

Coupled oscillators are much-studied but we tackle the problem of coupled identical additive networks capable of periodic behavior, not just simple oscillators, where we can examine the structure of the coupling itself in relation to the structure of the individual networks. This is a difficult question in general, but we restrict ourselves to the case of additive networks, weak connections, and most importantly from the point of view of tractability, to step function interactions. This last condition puts our networks into the class of 'Glass networks' [3] for which quite a bit of analysis has already been done. We will use some of this theory to get the easy part of our results, namely deducing from a stable periodic orbit for the uncoupled networks the existence of a synchronous periodic orbit for (weakly) symmetrically coupled networks that is stable to perturbations within the synchronous subspace. The harder part will be to find conditions on the cross connections between parallel networks that promote synchronization and make the synchronous