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A SIMPLE AND EFFICIENT METHOD FOR COMPUTING CENTER MANIFOLD AND NORMAL FORMS ASSOCIATED WITH SEMI-SIMPLE CASES

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Abstract. In this paper we present a simple and efficient method for computing center manifold and normal form of differential equations. The main attention is focused on general semi-simple singularities, that is, the Jacobian of a given system evaluated at an equilibrium contains semi-simple eigenvalues. The method combines the computation of center manifold and normal form in one unified procedure. The approach improves the existing general formulas and, in particular, leads to developing an efficient algorithm which directly yields the kth-order algebraic equations containing only the kth-order terms. The efficient method greatly saves computation time and reduces computer memory demanding. Symbolic programs using Maple have been developed, and applied to a physical oscillator model to show the efficiency of the new approach.
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1 Introduction

Normal form theory has been proved useful to transform a problem to a simpler form before trying to solve it. A lot of the early applications were in models from celestial mechanics. In the past two decades, normal form theory has been widely used in the study of complex behaviour of nonlinear dynamical systems such as bifurcation and instability (e.g., see [2, 5, 6, 11]). The basic idea of normal form theory is applying a series of near identity nonlinear transformations to systematically construct a simple form of the original system, which keeps the dynamic characteristics of the original system. Thus the analysis of the dynamical behaviour becomes simpler.

In general, with the conventional approach, a series of nonlinear transformations are introduced to reduce the original system to a locally invariant small dimensional manifold called center manifold. Then additional nonlinear transformations are applied to transform the center manifold to a simpler system called normal form. In the computation of normal forms the so-called non-resonant terms are removed using the nonlinear transformation while the resonant terms have to be retained as the normal form of the original system.