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CHAOTIC BEHAVIOUR OF A PREDATOR–PREY SYSTEM IN THE CHEMOSTAT

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Abstract. Generally a predator–prey system is modelled by two ordinary differential equations which describe the rate of changes of the biomasses. Since such a system is two-dimensional no chaotic behaviour can occur. In the popular Rosenzweig-MacArthur model, which replaced the Lotka-Volterra model, a stable equilibrium or a stable limit cycle exist. In this paper the prey consumes a non-viable nutrient whose dynamics is modelled explicitly and this gives an extra ordinary differential equation. For a predator–prey system under chemostat conditions where all parameter values are biologically meaningful, coexistence of multiple chaotic attractors is possible in a narrow region of the two-parameter bifurcation diagram with respect to the chemostat control parameters. Crisis-limited chaotic behaviour and a bifurcation point where two coexisting chaotic attractors merge will be discussed. The interior and boundary crises of this continuous-time predator–prey system look similar to those found for the discrete-time Hénon map. The link is via a Poincaré map for a suitable chosen Poincaré plane where the predator attains an extremum. Global homoclinic bifurcations are associated with boundary and interior crises.
Keywords. Bifurcation analysis - Chaos - Chemostat - Homoclinic bifurcation - Predator–prey system

1 Introduction

The dynamics of a predator–prey system is studied in many papers, for example by Rosenzweig and MacArthur [1] in an ecological context and under chemostat conditions by Nisbet *et al.* [2] in a microbiological context. In a minimal model, the state of each population is described by its size, that is the number of individuals or the population biomass, whereby no distinction between individuals is made. Then the dynamics of a population is mathematically described by an ordinary differential equation (ODE). So, with a predator-prey system its dynamical behaviour is described by a system of ODEs whereby the interactions between the populations and with the environment are taken into account.

The Rosenzweig-MacArthur model [1] which replaced the classical Lotka-Volterra is a predator–prey system where the prey grows logistically. In the