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A STOCHASTIC DIFFERENTIAL EQUATION MODEL FOR CHARGED-PARTICLE ENERGY STRAGGLING

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Abstract. For high-energy charged particles passing through matter, a stochastic differential equation is derived that approximately describes the rate of change of the particle energy with respect to distance. To study the accuracy of the stochastic differential equation model, an efficient Monte Carlo procedure based on sampling from the Landau distribution is used to provide reference results for the energy losses of charged particles passing through a medium. Calculational results indicate that the accuracy of the stochastic differential equation model increases as the thickness of the medium increases (i.e., as the energy loss increases).

Keywords. Straggling, Vavilov, Landau, stochastic differential equation, high-energy particles, nuclear physics, energy loss, Monte Carlo.
AMS (MOS) subject classification: 65C30, 65C05, 82C70.

1 Introduction

High-energy charged particles suffer collisions with electrons and lose energy as they pass through a medium [7,11]. Variability in this energy loss results from the random nature of the interactions. This variability is referred to as energy straggling [2,8,10]. It is assumed here that the rest mass of the charged particles is much greater than the rest mass of an electron and the energy of the charged particles is sufficiently high so that the particles do not capture electrons. With these assumptions, the charged particles lose energy by excitation and ionization with the atoms in the medium. The mechanism of the energy loss is primarily the interaction of the Coulomb fields of the particles with those of the bound electrons in the atoms of the medium [11].

If the energy loss ΔT is small compared to the initial energy T (i.e., $\Delta T/T < 0.1$), then the Vavilov distribution accurately describes the energy-loss distribution [1,8,9,10]. The Vavilov distribution, however, is quite complicated. As a result, the Vavilov distribution is difficult to work with and offers little insight into the energy-loss phenomenon. In addition, as the Vavilov distribution is valid for only small energy changes, a Monte Carlo calculational procedure is often used to estimate the energy losses for charged particles penetrating thick media [1,8].