Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **10 (2003)** 181-193

AN ALGORITHM FOR COMPUTING NORMAL FORMS USING PERTURBATION TECHNIQUE

Songhui Zhu and Pei Yu

Department of Applied Mathematics
University of Western Ontario, London, Ontario, Canada N6A 5B7

Abstract. A perturbation method based on multiple time scales is generalized to compute the normal forms of high dimensional oscillating systems. The particular attention is focused on autonomous systems. An algorithm for symbolic computation of the normal forms is presented and Maple programs have been implemented on computer systems. An example for a resonant case is given to demonstrate the applicability of the algorithm as well as the Maple programs.
Keywords. Autonomous system, normal form, multiple time scales, resonance, Maple.
AMS (MOS) subject classification: 34A34, 34C20, 68U99.

1 Introduction

Normal form theory is one of the useful and powerful tools in the study of nonlinear dynamical systems, in particular, for stability and bifurcation analysis (for example, see [3, 5, 6, 8]). Normal form is a simplified expression of the original system, obtained by applying successive transformations of the coordinates. The reduced system keeps the dynamic characteristics of the original system, and thus greatly simplifies the dynamical analysis.

This paper is focused on the study of nonlinear autonomous systems, which may be described by the general ordinary differential equation:

$$\dot{\boldsymbol{x}} = B\,\boldsymbol{x} + \boldsymbol{g}(\boldsymbol{x}), \quad \boldsymbol{x} \in R^n, \quad \boldsymbol{g}:\ R^n \to R^n \tag{1}$$

where B is an $n \times n$ constant matrix, and the function $\boldsymbol{g}(\boldsymbol{x})$ and all its first order partial derivatives are assumed to vanish at the origin, i.e., $\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0}$ and $\partial g_i(\boldsymbol{0})/\partial x_j = 0$ for $i, j = 1, 2, \ldots, n$. Here g_i and x_j are the ith and jth components of $\boldsymbol{g}$ and $\boldsymbol{x}$, respectively.

Assume a system has been reduced to its center manifold, described by equation (1). Then despite the low dimensionality of system (1), the analysis is still difficult, even if one is only interested in the qualitative behavior of the system. This is mainly due to the non-hyperbolicity assumed for the system, i.e., the linear part, $B\,\boldsymbol{x}$, of system (1) no longer determines the local dynamics as it does in the hyperbolic case. The problem is the intrinsic