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THE SIMPLEST NORMAL FORM FOR 1:2 DOUBLE HOPF SINGULARITY

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Abstract. This paper presents an attempt to generalize the computation of the simplest normal form of differential equations. The study is focused on the systems whose Jacobian evaluated at an equilibrium has 1:2 resonant double Hopf singularity. After obtaining the general conventional (classical) normal form, we have constructed the simplest normal form for the 1:2 resonance and proved that the pattern of the corresponding nonlinear transformation between the conventional normal form and the simplest normal form is the same as that of the conventional normal form. A simple, explicit recursive formula is derived for computing the simplest normal form of 1:2 resonance. The new recursive algorithm does not require for solving large matrix equations, but instead solving single linear algebraic equations one by one. In addition, unlike the conventional normal form theory which uses an independent nonlinear transformation at each order, this approach uses a consist nonlinear transformation through all order computations. The algorithm has been implemented on computer systems using Maple. The user-friendly programs can be executed to compute the coefficients of the simplest normal form and the associated nonlinear transformation.
Keywords. Differential equation, 1:2 resonance, conventional normal form, the simplest normal form, symbolic computation, Maple.
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1 Introduction

It is often proved useful to transform a problem to its simplest form before trying to solve it. Normal form theory is such an useful tool for computing a simpler form of the original differential equation but keep the qualitative dynamic behavior of the original system unchanged (e.g., see [4, 9, 10, 12, 15]). The general procedure of conventional (classical) normal form (CNF) theory is using the linear singularity of a system at an equilibrium to form a Lie bracket operator and then repeatly employing the operator to remove higher order nonlinear terms as many as possible. However, It is well known that CNF is by no means unique, and in fact further reductions on several cases of CNFs have been considered (e.g., see [2, 3, 5–8, 17]).

A further reduction of a CNF leads to the simplest normal form (SNF). From the computation point of view, in SNF theory, the kth-order nonlinear