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PALAIS-SMALE DECOMPOSITION LEMMAS IN AXIALLY SYMMETRY DOMAINS

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Abstract. In this article, we first establish a Palais Smale decomposition lemma, then apply it to assert the existence of solutions of equation (1) in $y-$symmetric unbounded domains.
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1 Introduction

Let $N \geq 2$ and $2 < p < 2^*$, where $2^* = \frac{2N}{N-2}$ for $N \geq 3$ and $2^* = \infty$ for $N = 2$. Consider the semilinear elliptic equation

$$\begin{cases} -\Delta u + u = |u|^{p-2}\, u \text{ in } \Omega, \\ u \in H_0^1(\Omega), \end{cases} \tag{1}$$

where Ω is a domain in $\mathbb{R}^N$ and $H_0^1(\Omega)$ is the Sobolev space in Ω. For the general theory of the Sobolev spaces $H_0^1(\Omega)$, see Adams [1]. Associated with equation (1), we consider the energy functionals a, b, and J for $u \in H_0^1(\Omega)$

$$\begin{aligned} a(u) &= \int_\Omega (|\nabla u|^2 + u^2), \\ b(u) &= \int_\Omega |u|^p, \\ J(u) &= \frac{1}{2}a(u) - \frac{1}{p}b(u). \end{aligned}$$

As in Rabinowitz [8, Proposition B. 10.], a, b, and J are of $C^{1,1}$. It is well$-$known that the solutions of equation (1) and the critical points of the energy functional J are the same. The existence of solutions of equation (1) which has been the focus of a great deal of research in recent years is affected by the shape of the domain Ω. For unbounded domains Ω, because the lack of compactness, the existence of solutions of equation (1) is not easy. In this article, we establish a decomposition lemma in Section 3 and then apply it to assert the existence of solutions of equation (1) in $y-$symmetric unbounded domains. In Section 2, we present lemmas to prove the decomposition lemma.

2 Preliminary

Let $z = (x, y) \in \mathbb{R}^{N-1} \times \mathbb{R}$. Denote the $N-$ball $B^N(z_0; s)$ in $\mathbb{R}^N$, the infinite strip $\mathbf{A}^r$, the upper half strip $\mathbf{A}_0^r$, and the finite strip $\mathbf{A}_{s,t}^r$ as follows: