Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 75-80

LIMITING BEHAVIOR OF A CLASS OF DELAY DIFFERENCE EQUATIONS

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Abstract. We study the limiting behavior of the difference equation $x_{n+1} = x_n + f(x_{n-k})$, where f satisfies the McCulloch-Pitts nonlinearity. This equation describes the dynamics of a single neuron with no internal decay and piecewise constant argument. It is shown that every solution tends to ∞ or $-\infty$ or is truncated periodic with the minimal period $2(2l+1)$ for some $l \geq 0$ such that $\frac{k-l}{2l+1}$ is a nonnegative odd integer. These results are different from those for $x_{n+1} = x_n - f(x_{n-k})$.
Keywords. Difference equation, Limiting behavior, Neural network, Truncated periodic solution, MuCulloch-Pitts nonlinearity, Piecewise constant argument
AMS (MOS) subject classification: 39A11

1 Introduction

Recently, more and more attention has been paid to the study of neural networks because of their wide and potential applications such as content-addressable memory, pattern recognition and so on. Usually, researchers use networks of small number of neurons, especially one or two, as prototypes to understand the dynamics of large scale networks. In the literature, the delay differential equation

$$\dot{x}(t) = f(x(t-\tau)) \tag{1}$$

is used as the model for a single neuron with no internal decay, where $f : \mathbb{R} \to \mathbb{R}$ is either the sigmoid or a piecewise linear signal function and $\tau \geq 0$ is the synaptic transmission delay (see, for example, [3, 6, 7, 9] and the references therein). If there exists $x_0 \in \mathbb{R}$ such that $(x - x_0)f(x) \geq 0$ (respectively, ≤ 0) then (1) is said to be with positive (respectively, negative) feedback.

As we know, differential equations with piecewise constant arguments have wide applications in certain biomedical models (see, [1], for instance). On the other hand, from the point of view of control, it is realistic to consider models with piecewise constant arguments. Therefore, we are inspired to propose

$$\dot{x}(t) = f(x([t])) \tag{2}$$

as a model for a single neuron with no internal decay, where $[\cdot]$ denotes the greatest integer function. In contrast to the assumptions on f for equation (1)