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PERIODIC TRAVELING WAVES IN REACTION DIFFUSION SYSTEMS WITH DELAYS

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Abstract. Existence of periodic traveling waves in systems of reaction diffusion equations with delays are studied. Instead of bifurcation approach which usually involves analyzing very complicated characteristic equations, a continuation theorem of coincidence degree theory is employed.

Key words. Reaction diffusion equations, traveling waves, functional differential equations, periodic solutions, coincidence degree.

AMS (MOS) Classification: 34K15, 34K20

1 Introduction

Consider the reaction diffusion system

$$\frac{\partial u(x,t)}{\partial t} = D\Delta u(x,t) + f(u(x,t)), \quad t \geq 0, \quad x \in \Omega \subset R^m. \tag{1.1}$$

where $u \in R^n$, $D = diag(d_1, d_2, \cdots, d_n)$ with $d_i > 0$, $i = 1, \cdots, n$, and Δ is the Laplacian operator with respect to the spatial variable x, that is,

$$\Delta u(x,t) = \left(\sum_{k=1}^{m} \frac{\partial^2 u_1(x,t)}{\partial x_k^2}, \cdots, \sum_{k=1}^{m} \frac{\partial^2 u_n(x,t)}{\partial x_k^2} \right)^T$$

Eq. (1.1) is an very important type of partial differential equations, for such equations have been used to model various problems arising from physics, chemistry and biology, etc. Among various aspects for (1.1), are the so-called traveling wave solutions, which are solutions of the form $u(x,t) = \phi(x \cdot \theta + ct)$, where $\theta = (\theta_1, \theta_2, \cdots, \theta_m)$ is a unit vector describing the direction of the wave and c is a constant giving the velocity of the wave. Fisher [3] and Kolmogonov et al. [7] first revealed that, like wave equations, a reaction diffusion equation can also allow traveling wave solutions. Since then, there have been many studies on traveling waves of reaction diffusion equations,