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OPTIMIZATION IN EIGENVALUE PROBLEMS

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Abstract. We study maxima and minima eigenvalues in a class of Sturm-Liouville problems. These problems arise in the investigation of the vibration of a string.

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1 Introduction

Let Ω be a bounded plane domain with a smooth boundary $\partial\Omega$ and let $D \subset \Omega$ be a measurable subset with $|D| = \beta$, $0 < \beta < |\Omega|$. For a fixed a real number α, let $\lambda = \lambda(\Omega, D, \alpha)$ be the first eigenvalue of the Dirichlet problem

$$-\Delta u + \alpha \chi_D u = \lambda u \quad \text{in} \quad \Omega, \quad u = 0 \quad \text{on } \partial\Omega. \tag{1.1}$$

If u is the first eigenfunction of problem (1.1) then

$$-\Delta u - \alpha \chi_{D^c} u = (\lambda - \alpha) u \quad \text{in} \quad \Omega, \quad u = 0 \quad \text{on } \partial\Omega, \tag{1.2}$$

where $D^c = \Omega \setminus D$. If $\alpha < 0$ then we can replace problem (1.1) by problem (1.2) with $-\alpha > 0$ and $|D^c| = |\Omega| - \beta$. Therefore, we confine our attention to problem (1.1) with $\alpha > 0$. Furthermore, we keep Ω and α fixed, so that $\lambda = \lambda_D$.

In [5,6] the following problem

$$\min_{D \subset \Omega, |D| = \beta} \lambda_D \,, \tag{1.3}$$

is investigated. First, an existence result is proved. Next, assuming Ω Steiner symmetric, the authors prove that for any minimizer D_m, its complement D_m^c is Steiner symmetric. As a consequence, if Ω is a disk, the uniqueness of a minimizer D_m follows. If Ω is symmetric with respect to a line, but it is not Steiner symmetric, results of symmetry breaking for D_m are proved. In these cases, there is not uniqueness of the minimizer. Also, in [5] and in [6], several results about the regularity of D_m are proved.

To motivate the work, in [5] the following physical model is described. If Ω is an elastic membrane made of two different materials D and D^c, equation