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COMPARISON PRINCIPLES FOR REACTION–DIFFUSION SYSTEMS WITH RESPECT TO PROPER POLYHEDRAL CONES

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Conditions on the Jacobian of the reaction term and the diffusion matrix are described such that the difference of solutions of a reaction–diffusion system remains in a cone if the corresponding difference of initial and boundary conditions is in the same cone. A further condition on the Jacobian implies that the difference of solutions stays in the interior of the cone. The sufficient conditions can be shown to be necessary. Applications of some of these theorems to reaction–diffusion systems modeling mass action chemical kinetics are presented.
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1 Introduction

Comparison of solutions to reaction–diffusion systems is usually based on arguments involving Maximum-Minimum Principles (see [9]) and are done with respect to the partial order coming from the nonnegative orthant $\mathbb{R}^n_+$. Similar results exist for ordinary differential equations systems [6, 12]. In this paper the comparison is done with respect to a partial ordering determined by a proper polyhedral cone K. The results of [4, 5] for an ordinary differential equations system are extended to reaction–diffusion system with an additional requirement coming from the introduction of the diffusion term.

The irreducibility of the Jacobian matrix of the reaction term implies that the difference of solutions in the ordinary differential equations case stays in the interior of $\mathbb{R}^n_+$, [12]. This result has been extended to reaction–diffusion systems by using K-irreducibility of the Jacobian matrix and likewise the difference of solutions stays in the interior of the cone K. The reader may consult [5] for the ordinary differential equations case.

Two example applications are considered: an elementary step reaction and part of the Belousov-Zhabotinsky reaction.