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EVENTUALLY PERIODIC SOLUTIONS OF $\mathbf{x_{n+1}} = \max\left\{\frac{\mathbf{A_n}}{\mathbf{x_n}}, \frac{\mathbf{B_n}}{\mathbf{x_{n-1}}}\right\}$ WHEN THE PARAMETERS ARE TWO CYCLES

C.M. Kent*, M. Kustesky, A.Q. Nguyen, and B.V. Nguyen

Department of Mathematical Sciences
Virginia Commonwealth University
Oliver Hall, 1015 West Main Street, P. O. Box 842014
Richmond, Virginia 23284-2014, USA
*e-mail: cmkent@saturn.vcu.edu

Abstract. We show that every positive solution of the difference equation

$$x_{n+1} = \max\left\{\frac{A_n}{x_n}, \frac{B_n}{x_{n-1}}\right\}, \quad n = 0, 1, \ldots$$

is eventually periodic with period four, where $\{A_n\}_{n=0}^{\infty}$ is a periodic sequence of positive real numbers with prime period two and $\{B_n\}_{n=0}^{\infty}$ is a periodic sequence of positive real numbers with period two.
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1 Introduction

We show that every positive solution of the difference equation

$$x_{n+1} = \max\left\{\frac{A_n}{x_n}, \frac{B_n}{x_{n-1}}\right\}, \quad n = 0, 1, \ldots \tag{1}$$

is eventually periodic with period four, where $\{A_n\}_{n=0}^{\infty}$ is a periodic sequence of positive real numbers with prime period two and $\{B_n\}_{n=0}^{\infty}$ is a periodic sequence of positive real numbers with (not necessarily prime) period two. We refer to difference equations involving the maximum function as "max equations."

The following is a summary of some of the results obtained so far on the eventual periodicity of solutions of max equations having the same form as Eq. (1) with the parameters held constant or allowed to vary periodically with various periods.