Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 463-476

SYNCHRONIZATION OF COUPLED EQUATIONS OF HODGKIN-HUXLEY TYPE

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Abstract. We study a class of differential equations modelling the electrical activity in biological systems. This class includes the Hodgkin-Huxley equations for the nerve impulse, as well as models for other excitable tissue, like muscle fibers, pacemakers and pancreatic cells .

We show that when two of these equations are coupled their solutions always synchronize. Synchronization takes place regardless of the initial condition if the coupling is strong enough, and even for two equations with different parameter values, coupled asymetrically.

We find a bounded region in phase space that attracts the flow globally and thus contains all points with recurrent behaviour. The size of the region can be calculated from the parameters in the equations. Thus we show that the system is uniformly dissipative. We obtain explicit bounds for this region in terms of the parameters as a tool for establishing synchronization. These estimates are also obtained for the uncoupled equations.

Keywords. Synchronization; Nerve impulse; Hodgkin-Huxley; Uniform dissipativeness; Attractor

1 Introduction

Many authors have observed numerically that when two equations similar to the Hodgkin-Huxley model are coupled, their solutions seem to synchronize. In some cases this is confirmed by a linear analysis near periodic orbits. These results were prompted by experimental findings of sychronization in excitable tissue. However, "mathematical models for these systems are typically very complicated and there has been very little rigorous analysis of them"[23]. We shall not review experimental and numerical results as this paper is concerned with an analytical treatment of synchronization.

We give here a proof that synchronization in partially coupled general equations of Hodgkin-Huxley type is a global behaviour for strong enough coupling. Synchronization takes place for asymetrically coupled equations with different parameter values. The result is analytical, as opposed to numerical simulations in the literature.