

OSCILLATION OF SECOND ORDER DIFFERENTIAL EQUATIONS WITH DAMPING

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Abstract. The purpose of this paper is twofold: to review recent results concerning oscillation of various classes of nonlinear second order differential equations with damping term and to present a new oscillation criterion for the general nonlinear differential equation with nonlinear damping term. The paper is concluded with the discussion of possible improvements and extensions of these results.

Keywords. Nonlinear differential equations, second order, damping term, proper solutions, oscillation, integral averaging method, interval method.

AMS (MOS) subject classification: 34C10.

1 Introduction

Very recently, there has been an increasing interest in obtaining sufficient conditions for oscillation of solutions for different classes of nonlinear second-order differential equations with damping term. This resulted in an impressive series of papers published during the last 15 years (to mention a few, we refer to the recent papers [1] - [18] where further references can be retrieved).

In the first part of this paper, we review principal results concerning oscillation of nonlinear differential equations with damping term of the form

$$(r(t)x'(t))' + p(t)x'(t) + q(t)f(x(t)) = 0 \quad (1)$$

and

$$(r(t)\psi(x(t))x'(t))' + p(t)x'(t) + q(t)f(x(t)) = 0, \quad (2)$$

which have been studied more intensively. We focus our attention on the basic assumptions and main techniques used for proving oscillation criteria, compare them, and analyze eventual advantages and disadvantages. A number of interesting results are stated without proofs and examples illustrating their relevance are provided.

We conclude the paper by presenting a new result obtained very recently by the authors for the general second order nonlinear differential equation with nonlinear damping term

$$(r(t)k(x(t), x'(t)))' + p(t)k(x(t), x'(t))x'(t) + q(t)f(x(t)) = 0, \quad (3)$$

and indicate the directions for further studies.