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AN EXPLICIT FORMULA OF THE BIFURCATION CURVE FOR A BOUNDARY BLOW-UP PROBLEM

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Abstract. We study the bifurcation curve of (sign-changing and nonnegative) solutions of the boundary blow-up problem

$$-\left(\varphi_p\left(u'(x)\right)\right)' = \lambda f_{a,b}(u(x)), \ \ -1 < x < 1,$$
$$\lim_{x\to 0^+} u(x) = \infty = \lim_{x\to 1^-} u(x),$$

where $p > 1$, $\varphi_p(y) = |y|^{p-2}\, y$ and $\left(\varphi_p(u')\right)'$ is the one-dimensional p-Laplacian, $\lambda > 0$, and

$$f_{a,b}(u) := \begin{cases} -u^a \text{ with } a > p-1 & \text{if } u \geq 0, \\ -k(-u)^b \text{ with } b > 0,\ k > 0 & \text{if } u < 0. \end{cases}$$

We give a simple explicit formula of the bifurcation curve of solutions. Thus we are able to determine the shape of the bifurcation curve and hence the exact multiplicity of solutions for any $\lambda > 0$. Moreover, when $b = a > p-1 > 0$, $k = 1$, and $f_{a,b}(u) = -|u|^a$, we investigate the variation of the bifurcation curves with respect to the exponent a.
Keywords. boundary blow-up problem, nonnegative solution, sign-changing solution, exact multiplicity, bifurcation curve, gamma function.
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1 Introduction

In this paper we study the bifurcation curve of (sign-changing and nonnegative) solutions of the boundary blow-up problem

$$\begin{aligned} -\left(\varphi_p\left(u'(x)\right)\right)' &= \lambda f_{a,b}(u(x)), \ \ -1 < x < 1, \\ \lim_{x\to 0^+} u(x) &= \infty = \lim_{x\to 1^-} u(x), \end{aligned} \tag{1}$$

where $p > 1$, $\varphi_p(y) = |y|^{p-2}\, y$ and $\left(\varphi_p(u')\right)'$ is the one-dimensional p-Laplacian, $\lambda > 0$ and

$$f_{a,b}(u) := \begin{cases} -u^a \text{ with } a > p-1 & \text{if } u \geq 0, \\ -k(-u)^b \text{ with } b > 0,\ k > 0 & \text{if } u < 0. \end{cases} \tag{2}$$

For $p = 2$, $\left(\varphi_p\left(u'(x)\right)\right)' = u''$, and (1) reduces to

$$-u''(x) = \lambda f_{a,b}(u(x)), \ \ -1 < x < 1,$$
$$\lim_{x\to 0^+} u(x) = \infty = \lim_{x\to 1^-} u(x).$$