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THE EVOLUTION OF REACTION-DIFFUSION WAVES IN GENERALIZED FISHER EQUATIONS: EXPONENTIAL DECAY RATES

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Abstract. In this paper, we consider an initial-boundary value problem for the generalized Fisher equation of order $m > 1$, with positive initial data being of $O(x^{-n}e^{-\sigma x})$ at large x (dimensionless distance), where $\sigma > 0$ and $-\infty < n < \infty$, are constants. Using the method of matched asymptotic expansions we are able to obtain the complete structure of the solution to the initial-boundary value problem when $\sigma > v^*(m)$ and $n < 0$ for large t (dimensionless time), which exhibits the formation of a permanent form reaction-diffusion wave (RDW) structure (which travels at the minimum available speed, $v = v^*(m)$). In particular, we determine the wave speed for the large-t RDW, the correction to the wave speed and the rate of convergence of the solution of the initial-boundary value problem onto the RDW. We conclude by stating a conjecture regarding the development of RDWs in the remaining cases.

Keywords. Initial-boundary value problem, generalized Fisher equation, matched asymptotic expansions, reaction-diffusion, permanent form travelling waves.
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1 Introduction

In this paper we consider the following initial-boundary value problem for the generalized Fisher equation, namely,

$$\left.\begin{array}{lll} u_t = u_{xx} + u^m(1-u), & x,t > 0, & \text{(P1)} \\ u(x,0) = u_0(x), & x \geqslant 0, & \text{(P2)} \\ u_x(0,t) = 0, & t > 0, & \text{(P3)} \\ u(x,t) \to 0 \text{ as } x \to \infty, & t \geqslant 0, & \text{(P4)} \end{array}\right\} \quad [\mathbf{P},\mathbf{m}]$$

where the reaction order m>1, and $u_0(x)$ is a continuous, analytic, positive and monotone decreasing function in $x \geqslant 0$, with $u_0(x) \to 0$ as $x \to \infty$. In particular, we suppose that,

$$u_0(x) \sim \left\{\begin{array}{ll} u_\infty x^{-n}e^{-\sigma x} + O[e^{-f(x)}] & \text{as } x \to \infty, \quad \text{(g1)} \\ \tilde{u}_0 + \sum_{l=1}^{\infty} \tilde{u}_l x^l & \text{as } x \to 0^+, \quad \text{(g2)} \end{array}\right.$$

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