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COMPLETE BLOW-UP OF SOLUTIONS FOR DEGENERATE SEMILINEAR PARABOLIC EQUATIONS

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Abstract. Let q be a nonnegative real number, f be a nonnegative smooth function, and t_b be a positive constant. This article studies the complete blow-up of weak solutions of the following degenerate semilinear parabolic problem,

$$\begin{aligned} x^q u_t - u_{xx} &= f(u),\ 0 < x < 1,\ 0 < t < t_b, \\ u(x,0) &= u_0(x),\ 0 \le x \le 1, \\ u(0,t) &= 0,\ u(1,t) = 0,\ 0 < t < t_b, \end{aligned}$$

where $u_0(x) \in C^{2+\alpha}([0,1])$ for some constant $\alpha \in (0,1)$ is a nontrivial and nonnegative function such that $u_0'' + f(u_0) \ge 0$ in $(0,1)$, and $u_0(0) = u_0(1) = 0$. Two criteria for weak solutions to blow up completely in the interval $(0,1)$ at t_b are given.
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1 Introduction

Let q be a nonnegative real number, f be a nonnegative smooth function, t_b be a positive constant, $D = (0,1)$ and $\bar{D} = [0,1]$. We consider the following degenerate semilinear parabolic first initial-boundary value problem,

$$x^q u_t - u_{xx} = f(u) \text{ in } D \times (0, t_b) \tag{1}$$

$$u(x,0) = u_0(x) \ge 0 \text{ on } \bar{D},\ u(0,t) = 0 = u(1,t) \text{ for } 0 < t < t_b, \tag{2}$$

where $u_0(x) \in C^{2+\alpha}(\bar{D})$ for some constant $\alpha \in (0,1)$ is a nontrivial and nonnegative function such that $u_0''(x) + f(u_0) \ge 0$ in D, and $u_0(0) = 0 = u_0(1)$. Since x^q tends to zero as x approaches 0, (1) is said to be degenerate.