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QUENCHING AND BLOW-UP PHENOMENA

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Abstract. Quenching and blow-up phenomena modeled by first initial-boundary value problems involving degenerate semilinear parabolic equations are studied. Quenching in a finite time, quenching in infinite time, locations of quenching points, impulsive quenching, and beyond quenching are discussed. Blow-up, impulsive blow-up, complete blow-up, and single-point blow-up are considered.
Keywords. Quenching, beyond quenching, critical length, blow-up, concentrated nonlinear source, nonlinear source of local and nonlocal features.
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1 Introduction

Quenching and blow-up problems occur in many important applications in science and engineering, for examples, the Langmuir-Hinshelwood model in heterogeneous chemical catalyst kinetics (cf. Aris [1], and Diaz [21]), the enzyme-kinetics model (cf. Banks [2], and Diaz [21]), the solid fuel ignition (cf. Bebernes and Eberly [3]), avalanches, fluid dynamics (cf. Floater [22]), the thermal explosion (cf. Chan and Kong [9]), the channel flow (cf. Chan and Kong [11]), nuclear reactor kinetics (cf. Pao [27]), and the damage of materials (cf. Chan and Ke [6]).

Let q and a be constants such that $q \geq 0$ and $a > 0$, $T \leq \infty$, $D = (0, a)$, $\Omega = D \times (0, T)$, $\bar{D}$ be the closure of D, and $L = x^q \partial/\partial t - \partial^2/\partial x^2$. This article discusses degenerate semilinear singular parabolic first initial-boundary value problems of the following type:

$$\left.\begin{array}{c} Lu = f(u) \text{ in } \Omega, \\ u(x,0) = u_0(x) \text{ on } \bar{D}, \\ u(0,t) = u(a,t) = 0 \text{ for } 0 < t < T. \end{array}\right\} \qquad (1)$$

When $q = 0$, L is the heat operator. When $q = 1$, L occurs in the model, studied by Ockendon [24], for the flow in a channel of a fluid whose viscosity in the boundary layer depends on the temperature u with x and t denoting, respectively, the coordinates perpendicular and parallel to the channel walls.