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EXISTENCE OF MONOTONE TRAVELING FRONTS FOR BDF DISCRETIZATIONS OF BISTABLE REACTION-DIFFUSSION Equations

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Abstract. This article is concerned with the effect of temporal discretization on traveling wave solutions to parabolic PDEs possessing bistable nonlinearities. The focus is on the application of backward differentiation formulas to Nagumo type PDEs with two different bistable nonlinearities. Existence of monotone traveling fronts is shown and the efficacy of different methods of proof is discussed.

Keywords. bistable partial differential equation, traveling waves, backward differentiation formulas.

AMS (MOS) subject classification: 35K57, 73D99, 65L, 65M

1 Introduction

Nonlinear bistable parabolic partial differential equations are commonly used in the modeling of phase transitions. These types of PDEs are often referred to as bistable equations, Allen-Cahn equations, and Chafee-Infante equations. Another example where the depths of the potential wells are allowed to vary is the Nagumo [26] PDE. The Nagumo equation admits monotone traveling wave solutions that are stable and attract a large class of initial conditions (see [2] and [11]). When these types of equations are solved numerically (albeit on a truncated finite spatial domain), software codes such as LSODE [16], which are based upon backward differential formulas (BDF), are often employed. These BDF methods are appropriate for stiff differential equations and the stable BDF methods [13] are so called $A(\alpha)$ stable methods.

Our contribution in this paper is to consider BDF temporal discretizations of Nagumo partial differential equations with two bistable nonlinearities and show the existence of monotone traveling fronts. The two nonlinearities considered are a piecewise linear nonlinearity, that has been considered by McKean [27] among others, and a cubic nonlinearity. The existence of monotone traveling waves for the piecewise linear nonlinearity involves the use of transform techniques and follows results in [27], [6], [7]. Due to lack of smoothness the only member of the BDF family that we are able to prove the existence of a monotone traveling front is the backward Euler method. For