Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 373-387

ON KERNEL ESTIMATIONS AND INVARIANT MEASURES OF STOCHASTIC JUMP-DIFFUSIONS

C. W. Li[1] and Z. Dong[2]

[1]Department of Mathematics
City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong
E-mail: macwli@cityu.edu.hk
[2]Institute of Applied Mathematics
Academy of Mathematics and Systems Sciences
Academia Sinica, Beijing 100080, P. R. China

Abstract. Under certain assumptions on the jumps, we give both the lower and upper heat-kernel bounds for the stochastic jump-diffusion processes with the diffusion part in the divergence form. We also study the existence of invariant probability measures of stochastic jump-diffusions on $\mathbb{R}^d$ and they have relations to those of the corresponding diffusions. In addition the ergodic property of invariant probability measure is investigated by use of the heat-kernel bound.
Keywords. Jump-diffusion, heat-kernel bound, Lyapunov function, invariant measure, ergodicity.
AMS (MOS) subject classification (2000): Primary: 58J65, 28C10; Secondary: 65C30, 37A25, 37B25.

1 Introduction

The estimation of transition probability densities and the existence of invariant probability measures are very important in the stochastic analysis. There is a lot of works in this aspect for diffusion processes. One of the well-known estimation is the heat-kernel bound for the differential operators in the divergence form, see Aronson [1], Davies [4] and Stroock [10] for an introduction. Recently the smoothness of transition probability densities for jump processes has been studied, see Picard [7] for the pure jump case and Bichteler, Gravereaux and Jacod [3] for the jump-diffusion case by the aid of the Malliavin calculus. Existence of invariant probability measures always involves the stability of the stochastic processes. It can be applied to the ergodic theorem and the representation of the top Lyapunov exponents, see Baxendale [2] for applications in diffusions.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, I\!\!P)$ be the underlying filtered complete probability space. Consider the following system of stochastic differential equation with jumps,

$$dx(t) = a(x(t))dt + b(x(t))dW(t) + \int_Z c(x(t-), z)N(dt, dz), \qquad (1.1)$$