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ON THE STUDY OF SINGULARITIES FOR A PLANAR SYSTEM WITH TWO DELAYS

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Abstract. For an equilibrium point of a two dimensional functional differential equation with two delays it is shown under which conditions Hopf and Bogdanov-Takens singularities occur. For the latter case, the Bogdanov-Takens bifurcation is completely described by introducing suitable bifurcation parameters.

Keywords. Bogdanov-Takens singularity, Hopf singularity, Bogdanov-Takens bifurcation, normal form, center manifold.

AMS (MOS) subject classification: 34K17, 34K18, 34K20

1 Introduction

We consider the planar system

$$\begin{cases} \dot{x}_1(t) = f_1(x_1(t), x_2(t-\tau_2)) \\ \dot{x}_2(t) = f_2(x_1(t-\tau_1), x_2(t)) \end{cases} \tag{1.1}$$

where one of the delays τ_1, τ_2 is positive and the other non-negative, and $f_1, f_2 : \mathbb{R}^2 \longrightarrow \mathbb{R}$ are C^k functions, $k \geq 2$. System (1.1) includes a significant number of models in population dynamics, like predator-prey systems, as well as systems modeling a two neuron network without self-connections.

There is an extensive literature dealing with functional differential equations (FDEs) that can be reduced to form (1.1). In the case of predator-prey models with two delays, questions such as local and global stability of equilibria, persistence, existence of periodic orbits have been widely addressed by many authors. The aim of the present paper is to establish the singularities that might exist for (1.1), and to completely describe the case of the Bogdanov-Takens bifurcation that occur under some contraints on the partial derivatives of f_1, f_2 at an equilibrium. For equations similar to (1.1), there is a significant number of papers where the analysis of Hopf bifurcation has been carried out. To the best of our knowledge, for planar system the Bogdanov-Takens bifurcation has been studied only recently by Xiao and Ruan [7], for a particular case of (1.1) with only one delay, and by Hale and Tanaka [5], where an FDE in $\mathbb{R}^2$ with two delays is considered. (Note that the general case for scalar FDEs was previously studied in [2].)