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A REVIEW OF QUENCHING RESULTS IN THE CONTEXT OF NONLINEAR VOLTERRA EQUATIONS

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Abstract. This paper presents recent progress in the analysis of quenching phenomena for problems that lend themselves to analysis in the context of nonlinear Volterra integral equations. This work is placed in the broader context of quenching research that has been an active area of research for over twenty-five years.
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1 Introduction

The study of quenching behavior for nonlinear initial boundary value problems (IBVPs) has been an active area of research since H. Kawarada introduced the concept with the following problem in 1975 [38]. The phenomenon of quenching, where the solution of the problem remains bounded while the first order time derivative becomes unbounded in finite time, is discussed here in a survey-format. There is a special emphasis on recent contributions from problems that lend themselves to analysis in the context of nonlinear, Volterra integral equations. Consider the one-dimensional parabolic problem given by

$$Hv = (1-v)^{-1}, \quad 0 \le x \le l, \quad 0 \le t, \tag{1}$$

$$v(0,t) = v(l,t) = 0, \quad 0 \le t, \tag{2}$$

$$v(x,0) = 0, \quad 0 \le x \le l, \tag{3}$$

where $Hv \equiv v_t - v_{xx}$. (This notation for the heat operator will be used throughout this paper.) Kawarada determined a critical length $l_0 = 2\sqrt{2}$ such that if $l > l_0$, then $\lim_{t \to T^-} v(\frac{l}{2}, t) = 1$ for some $T < \infty$. Furthermore, he showed that as v reaches 1 (the quenching value), $\lim_{t \to T^-} \max_x v_t(x,t) = \infty$. Hence, the solution will *quench* in finite time. Several authors have since