

LOWER AND UPPER DIRECTIONAL DIFFERENTIABILITY OF THE SET VALUED MAPS AND THEIR APPLICATIONS

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Abstract.

In this paper the lower and upper directional differentiability concepts of the set valued maps are introduced and the directional differentiability properties of the marginal functions are studied. Sufficient condition ensuring the existence of the directional derivative of the marginal function is obtained.

Keywords: Set valued map, derivative sets and differential, marginal function

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1 Introduction

In the paper the lower and upper directional differentiability concepts of the set valued maps are introduced and the directional differentiability properties of the marginal functions are studied. Note that the marginal functions come into being in various problems of the applied mathematics, including the parametric optimization, differential games, optimal control problems (see, e.g. [4, 11, 13, 14]).

The marginal functions are not usually differentiable. But in some problems it is necessary that the directional derivative or the directional lower and upper derivatives of the marginal functions be calculated.

The directional derivatives of the marginal functions was studied in [3, 6-8, 10, 12]. When the marginal function is given by maximum or minimum of a finite number of differentiable functions, the directional derivatives of this function had been investigated in [6, 7]. The directional derivative of locally Lipschitz function is considered in [3].

In this paper the directional upper and lower derivative of the marginal functions are given using the concepts of the directional upper and lower differentials of the set valued map which defines the marginal function and sufficient condition ensuring the existence of the directional derivative of the marginal function is obtained.