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DEGENERATE HOPF BIFURCATION FOR QUASILINEAR DIFFERENTIAL ALGEBRAIC EQUATIONS

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Abstract. Consider the following quasilinear differential-algebraic equations

$$A(\mu, x)\dot{x} = G(\mu, x),$$

where $x \in \mathbb{R}^n$, μ is a real parameter, $A : I{\times}U{\to}\mathcal{L}(\mathbb{R}^n)$ and $G : I{\times}U{\to}\mathbb{R}^n$ are $C^k, k \geq 1$, $I{\subset}\mathbb{R}$ is an open interval and $U \subset \mathbb{R}^n$ is open. In addition assume $A(\mu, x)$ has constant but not full rank and then the system cannot be made into an explicit ODE. A generalization of the degenerate Hopf bifurcation theorem for the quasilinear differential-algebraic equations is proven under some degenerate condition i.e. the non-transversality condition, which is an extension of the result by Rabier in 1999.
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1 Introduction

Hopf bifurcation and degenerate Hopf bifurcation take place in a one-parameter family of ODE in $\mathbb{R}^n$

$$\dot{x} = F(x, \mu), \tag{1.1}$$

as the parameter $\mu \in \mathbb{R}$ varies if system (1.1) satisfies:

(i) $F(0, \mu) \equiv 0, F(0, 0) = 0;$

(ii) $D_xF(0, 0)$ has only a pair of pure imaginary eigenvalues on the imaginary axis.

In 1942, Hopf proposed the first general mathematical formulation of Hopf bifurcation [12], although in a special case. In 1970's, Hopf's result was then refined and generalized in various directions: Crandall and Rabinowitz [4] proposed an infinite dimensional version by Lyapunov-Schmidt reduction and highlighted the importance of the so-called non-resonance condition. Golubitsky and Langford [7] made used of singularity theory to analyze the situation where transversality condition of the classical Hopf bifurcation fails and gave a complete classification of Hopf bifurcation and degenerate Hopf bifurcation problems. Numerous other authors have contributed to our current understanding of Hopf bifurcation and degenerate Hopf bifurcation and a vast literature on this topic is available today.