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DETERMINISTIC AND STOCHASTIC DYNAMICS WITH HYPERBOLIC HJB-TYPE EQUATIONS

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Abstract. The study of deterministic and stochastic dynamic problems in control theory can be reduced to partial differential equations by using Bellman's approach. The resulting PDE-based mathematical models are difficult to solve numerically, and most existing approaches cannot provide accuracy expected in practical applications. It is proposed to approximate the original controlled dynamics with a sequence of PDEs. By applying the Steklov-Poincare operator technique the general form of equations in such a sequence has been established. The derived model can be used in cases that are not covered by standard diffusion processes.
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1 Introduction

Dynamics, both deterministic and stochastic, should be controlled in many applications ranging from industrial processes to problems in life sciences and financial market (e.g., [16, 8, 27]). The problem of control can be associated with a Hamilton-Jacobi-Bellman (HJB) equation, and with fundamental advances in viscosity solution theory and nonsmooth analysis there is a substantial body of literature in this direction [32, 29, 26, 18, 17, 2, 3, 6, 11, 12, 13, 5, 1]. One of the most important practical reasons for this interest to the HJB equation lies with the fact that when the value function is smooth, apart from giving the classical solution of that equation, the derivatives of this function allow us to construct optimal strategies in feedback form. It is well known, however, that the value function should be considered in some generalised sense (e.g., as contingent solution, semicontinuous viscosity solution, minimax solution, etc [5, 27]). In addition, the task of solving numerically the resulting equation remains extremely difficult and most available methodologies are yet to be improved in terms of accuracy in order to be applicable in realistic practical situations [17]. The problem becomes even much more difficult when control is a function of both temporal and spatial variables.

We recall that the coupling between position $x \equiv q$ and momentum p is an essential ingredient of both deterministic and stochastic dynamics. In