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GLOBAL ATTRACTOR FOR APPROXIMATE SYSTEM OF CHEMOTAXIS AND GROWTH

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Abstract. In [9], Mimura and Tsujikawa presented a nonlinear diffusion system of equations to describe the aggregating pattern formation by chemotaxis and growth. Some mathematical aspects of their equations have been studied in [11,12,13]. This paper is then concerned with an approximate system in a finite dimensional space. We shall construct a global attractor for this approximate system.

Keywords. approximate system, chemotaxis-growth, global attractor.
AMS (MOS) subject classification: 34D45, 65M60, 92C15

1 Introduction

We are concerned with the following initial value problem for a chemotaxis-growth system of equations

$$\begin{cases} \dfrac{\partial u}{\partial t} = a\Delta u - \nabla \cdot \{u \nabla \chi(\rho)\} + f(u) \quad \text{in} \quad \Omega \times (0,\infty), \\ \dfrac{\partial \rho}{\partial t} = b\Delta \rho - c\rho + du \quad \text{in} \quad \Omega \times (0,\infty), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial \rho}{\partial n} = 0 \quad \text{on} \quad \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x), \quad \rho(x,0) = \rho_0(x) \quad \text{in} \quad \Omega. \end{cases} \tag{CG}$$

This problem arises in biology, where $u(x,t)$ and $\rho(x,t)$ denote the population density of biological individuals and the concentration of chemical substance at a position $x \in \Omega \subset \mathbb{R}^2$ and time $t \in [0,\infty)$, respectively. The mobility of individuals consists of two effects: one is random walking, and the other is the directed movement in a sense that they have a tendency to move toward higher concentration of the chemical substance. This is called chemotaxis in biology [1,3,5,18]. $a > 0$ and $b > 0$ are the diffusion rates of u and ρ, respectively. $c > 0$ and $d > 0$ are the degradation and production rates of ρ, respectively. $\chi(\rho)$ is the sensitivity function due to chemotaxis. $f(u)$ is a growth term of u.

In order to study aggregating patterns due to chemotaxis and growth, there are several contributions not only from experiments but also from mathematical analysis. Budrene and Berg [4] experimentally observed that bacteria called *E. coli* form complex spatio-temporal colony patterns. In order to