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THE BERRY-ESSEEN THEOREM FOR SHORT DISTANCES UNDER WEAK DEPENDENCE

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Abstract. We prove the Berry-Esseen theorem of the empirical process of a_n close pairs for a strictly stationary ψ-mixing sequences. The result generalizes the results in [2].
Keywords. Berry-Esseen theorem, close pair, ψ-mixing, m-dependent, degenerate U-statistic.
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1 Introduction and the result

Let $\{\xi_i\}$ be a strictly stationary sequence of random vectors in R^m, with distribution function F and the density function f, satisfying the ψ-mixing condition

$$\psi(n) = \sup_{A\in\mathbf{M}_{-\infty}^0, B\in\mathbf{M}_n^\infty} \frac{|P(AB)-P(A)P(B)|}{P(A)P(B)} \downarrow 0, \ ((\mathbf{M}_a^b = \sigma(\xi_1,\cdots,\xi_b)).$$

Let the mapping $d: \mathsf{R}^m \times \mathsf{R}^m \to [0,\infty)$ have the following properties:

$$\begin{aligned}
&d(x,y) = d(y,x) \quad \text{for all } x,y \in \mathsf{R}^m;\\
&d(0,\lambda x) = |\lambda| d(0,x) \quad \text{for all } x \in \mathsf{R}^m \text{ and } \lambda \in \mathsf{R};\\
&d(x,y) = d(x+z,y+z) \quad \text{for all } x,y,z \in \mathsf{R}^m;\\
&{}^\exists C_1\,(>0)^\exists C_2\,(>0):\\
&\quad C_1\|x-y\| \le d(x,y) \le C_2\|x-y\| \quad \text{for all } x,y \in \mathsf{R}^m,
\end{aligned}$$

where $\|x\| = \max(|x_1|,\cdots,|x_n|)$ $(x=(x_1,\cdots,x_m) \in \mathsf{R}^m)$.

Let $\{a_n\}$ be a sequence of positive numbers such that $a_n \downarrow 0$ and define the empirical distribution function

$$S_n(t) = \frac{2}{n(n-1)} \sum_{1\le i<j\le n} I(d(\xi_i,\xi_j) \le t a_n), \quad 0 \le t \le 1.$$

Assuming that $\xi_1, \xi_2, \cdots$ are independent identically distributed random vectors, Eberl and Hafner [3] as well as Silverman and Brown [7] proved that, for fixed t, $S_n(t)$ converges to a Poisson random variable if $a(n) = n^{-2/m}$ and Kester [6], Weber [8] and Jammalamadaka and Janson [5] showed that