Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 289-298

ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF A HYPERBOLIC FREE BOUNDARY PROBLEM

Kazuaki Nakane[1] and Tomoko Shinohara[2]

[1]Depertment of Engineering, Osaka Institute of Technology, Asahi-ku, Osaka, Japan, 535–8585
[2]Graduate School of Natural Science and Technology, Kanazawa University Kakuma, Kanazawa, Japan, 920–1192

Abstract. A one-dimensional free boundary problem of hyperbolic type will be analyzed. This problem arises from the physical model " Peel a thin film from a domain". The local solutions have been already given for our problem. However, it is not sure that the solutions can be extended globally. In this article, by applying an iteration method, it will be shown that time-global solutions exist.
Keywords. free boundary, hyperbolic equation, peeling phenomena, variational problem, global existence.
AMS (MOS) subject classification: Primary 35R35;Secondary 35L70.

1 Introduction

Let us consider the following one-dimensional free boundary problem

$$(P)\quad \begin{cases} u_{xx} - u_{tt} = 0 & \text{in} \quad (0,\infty) \times \{t>0\} \cap \{u>0\}, \\ u_x^2 - u_t^2 = Q^2 & \text{on} \quad (0,\infty) \times \{t>0\} \cap \partial\{u>0\}, \end{cases}$$

with the initial conditions

$$(I)\quad \begin{cases} u(x,0) = e(x) & \text{in} \quad (-l_0, 0), \\ u_t(x,0) = g(x) & \text{in} \quad (-l_0, 0), \end{cases}$$

and the boundary condition

$$(B)\quad u(-l_0, t) = f(t) \qquad \text{for} \quad t \geq 0,$$

where $e(x)$, $g(x)$ and $f(t)$ are given functions, Q and l_0 are positive constants.

This problem arises from the following variational problem which is related to a physical model "Peel a thin film from a domain Ω "(see [5; Appendix]): The shape of a film is described by the graph of a function $u : \Omega \to \mathbf{R}$. Find a stationary point of the functional

$$J(u) := \int_0^{T^*} \int_\Omega \left(\frac{\tau}{2} |\nabla u|^2 - \frac{\rho}{2} (D_t u)^2 \chi_{u>0} + \frac{Q^2}{2\tau} \chi_{u>0} \right) dxdt \qquad u \in \mathrm{K} \tag{1.1}$$