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APPROXIMATION OF ORDINARY DIFFERENTIAL EQUATIONS WITH IMPULSES

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Abstract. A survey of methods based on fixed mesh variational formulations for ordinary differential equations in presence of a possibly infinite number of impulses on the righthand side is presented. Existence and uniqueness results for the solution and approximation schemes with their error estimates are obtained.
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1 Introduction

The object of this paper is to present fixed mesh variational formulations for ordinary differential equations (ODE) in presence of a possibly infinite number of impulses on the righthand side. These formulations lead to existence and uniqueness results for the solution of the impulsive ODE and to approximation schemes with error estimates.

The system considered is of the form

$$\begin{cases} \dot{x}(t) = f\big(x(t),t\big) + \sum\limits_{j\in J} \alpha_j\big(x(\tau_j^-)\big)\delta(t-\tau_j), & t \in [0,T], \\ x(0) = x^0, \end{cases} \tag{1}$$

where $T > 0$ is a real number, $x^0 \in E$ is the initial condition, $x : [0,T] \to E$ is a vector function, $f : E \times [0,T] \to E$ is a given map, $\delta(\cdot)$ is the Dirac delta function (distribution) in 0, J is a countable set of indices (a subset of $\mathbb{N}$), and for any $j \in J$, $\tau_j \in]0,T]$ and $\alpha_j : E \to E$ is a given map. Moreover, the sequence $\{\tau_j\}_{j\in J}$ is a strictly increasing sequence.

Under appropriate assumptions, the method of Carathéodory [1] for ODE ($J = \emptyset$) can be adapted and used to show that (1) has a unique solution x in an appropriate weak sense. It is made up of an absolutely continuous part