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DECAY OF MOTION AND ENERGY DIFFUSION IN A NONLINEAR LATTICE

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Abstract. We investigate the dynamics of a one-dimensional lattice where the particles interact with a potential proportional to the power of $\gamma + 1$ ($\gamma > 1$) by approximating it with a pair of conservation laws. The pair is analyzed by the viscosity method. It is proved that the viscosity equation has a global solution for the initial-boundary value problem and the solution decays with time t uniformly. The numerical analysis for $\gamma = 3$ and for a range of the viscosity coefficient ϵ showed that the solution diffuses in the form similar to a shock wave and that it can be analyzed in terms of the Rankine-Hugoniot jump condition. Its speed is proportional to t^s, $s = 0.5824$. For the smaller values of ϵ, the solutions oscillate with x. The speed of diffusion increases with the decrease of ϵ and in proportion to t. The kinetic plus dissipated energy is conserved.

Keywords. nonlinear lattice, conservation laws, viscosity method, energy diffusion.

AMS (MOS) subject classification: 35K57, 35L65, 70K75

1 Introduction

In order to investigate the physical features which may exist in the dynamics of nonlinear lattices, we consider the simple model expressed by the following Hamiltonian $\mathcal{H}_{N,\gamma}$ where the particles interact with the potential proportional to the power of $\gamma + 1$ ($\gamma > 1$) and with no linear term;

$$\mathcal{H}_{N,\gamma} = \sum_{i=0}^{N} \left[\frac{1}{2} \left(\frac{dU_i}{dt} \right)^2 + \frac{1}{\gamma(\gamma+1)a^{\gamma+1}} |U_{i+1} - U_i|^{\gamma-1} (U_{i+1} - U_i)^2 \right], \quad (1)$$

$$U_0 = U_{N+1} = 0,$$

where N is the number of particles in the lattice, a is the lattice constant, U_i is the displacement of the i-th particle from its equilibrium position and t is time.

This model relates with interesting problems. For the odd number of N and the odd integer of γ larger than three, Yoshida [8] and Umeno [7] showed that (1) has not the first integral other than Hamiltonian, so that it is not integrable. Kawai *et al* found by the numerical experiments on the case $\gamma = 3$ that the power spectrum of the phonon number fluctuation shows the $1/f$ type distribution on frequency f and that the trajectory spends equal