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QUASI-HOMOGENEOUS NORMAL FORMS FOR NULL LINEAR PART

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Abstract. In this paper, we analyze simplified normal forms for equilibria of vector fields with zero linear part, assuming the presence of a symmetry. We put in correspondence the vector field with zero linear part with a Takens-Bogdanov singularity with an invariant axis, which we study by using expansions in quasi-homogeneous components of a fixed type. We perform both, near-identity transformations as well as reparametrizations in the time to determine the structure of the normal form. A degenerate case, in which a normal form coefficient vanishes, is also included.
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1 Introduction

The main goal of this paper is to determine the structure of normal forms for planar vector fields with an equilibrium point at the origin with zero linear part, in the form:

$$\begin{aligned} \dot{u} &= auv + f(u,v), \\ \dot{v} &= bu^2 + cv^2 + g(u,v), \end{aligned} \tag{1}$$

where f, g are smooth functions that vanish, together with their derivatives up to second order, at the origin. Moreover, we assume that a symmetry:

$$u \longrightarrow -u,$$

is present. In particular, functions f, g must satisfy $f(-u,v) = -f(u,v)$, $g(-u,v) = g(u,v)$. Through this paper, we will consider formal expansions for functions f, g in the form:

$$f(u,v) = \sum_{i+j\geq 3,\ i=\text{odd}} a_{ij}u^i v^j, \quad g(u,v) = \sum_{i+j\geq 3,\ i=\text{even}} b_{ij}u^i v^j. \tag{2}$$

This case arises in the study of the Hopf-zero bifurcation, after dropping the azimuthal component in cylindrical coordinates. Namely, the normal