Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 233-245

A CLASS OF DIFFERENTIAL OPERATORS BASED ON $(\theta + zD)D + \omega zD$

Yuri Kozitsky[1], Piotr Oleszczuk [1] and Lech Wołowski[2,1]

[1]Instytut Matematyki, Uniwersytet Marii Curie-Skłodowskiej
PL 20-031 Lublin (Poland)
e-mail: jkozi@golem.umcs.lublin.pl
e-mail: poleszcz@golem.umcs.lublin.pl

[2]Department of Mathematics, University of California
Davis, CA 95616-8633, USA
e-mail: wolowski@math.ucdavis.edu

Abstract. Differential operators $\varphi(\Delta_{\theta,\omega})$ of infinite order, where φ is an exponential type entire function of a single complex variable and $\Delta_{\theta,\omega} = (\theta+\omega z)D + zD^2$, $D = \partial/\partial z$, $z \in \mathbb{C}$, $\theta \geq 0$, $\omega \in \mathbb{R}$, acting in the spaces of exponential type entire function are studied. For $\omega \geq 0$, such operators preserve the set of Laguerre entire functions provided the function φ also belongs to this set. The latter consists of the polynomials possessing real nonpositive zeros only and of their uniform limits on compact subsets of the complex plane $\mathbb{C}$. The operator $\exp(a\Delta_{\theta,\omega})$, $a \geq 0$ is studied in more details. In particular, it is shown that it preserves the Laguerre entire functions for all $\omega \in \mathbb{R}$. An integral representation of $\exp(a\Delta_{\theta,\omega})$, $a > 0$ is obtained. These results are used to obtain the solutions to certain Cauchy problems employing $\Delta_{\theta,\omega}$.
Keywords. Fréchet spaces; exponential type entire functions; Laguerre entire functions; nonpositive zeros; integral representation; Cauchy problem.
AMS (MOS) subject classification: 47E05, 46E10, 34M05, 30D15

1 Introduction

In this article we consider differential operators (including of infinite order) on the following differential expression

$$\Delta_{\theta,\omega} = \Delta_\theta + \omega z D \overset{\text{def}}{=} (\theta + zD)D + \omega z D, \tag{1.1}$$

where $D = \partial/\partial z$ stands for differentiation with respect to a single complex variable $z \in \mathbb{C}$ and $\theta \geq 0$, $\omega \in \mathbb{R}$ are its parameters.

Given entire functions $\varphi, f : \mathbb{C} \to \mathbb{C}$, we set

$$\left(\varphi(\Delta_{\theta,\omega})f\right)(z) = \sum_{k=1}^{\infty} \frac{1}{k!}\varphi^{(k)}(0)\left(\Delta_{\theta,\omega}^k f\right)(z). \tag{1.2}$$

In order for the above series to converge to an entire function we impose growth restrictions on the functions φ and f by placing them into certain