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TWO APPLICATIONS OF HYPOELLIPTICITY THEOREMS

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Abstract. We give applications of hypoellipticity theorems in the theory of partial differential equations. One is to prove hypoellipticity for the operator $x^q\partial_t - \partial_{xx}$. This degenerate parabolic operator appears in a model of quenching phenomena, studied widely as a pipe-cleaning problems in a field of applied mathematics. Another is to prove a regularity of solutions of functional equations of a generalized meanvalue type.
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1 Introduction

Hypoellipticity ia an extended notion of ellipticity. Roughly speaking, when an operator P is elliptic partial differential operator and equation $Pu = f$, if f is analytic then u is analytic. On the other hand an operator P is said to be hypoelliptic if f is of C^∞, then u is of C^∞.
A degenerate parabolic problem with nonlocalboudary conditions is stated as the following:

$$\begin{aligned} x^q u_t - u_{xx} &= f(u) \text{ in } (0,a)\times(0,T) \qquad (1.1)\\ u(x,0) &= 0, \text{ on } [0,a] \end{aligned}$$

$$u(0,t) = \int_0^a N(x)|u(x,t)|dx, \qquad u(a,t) = \int_0^a M(x)|u(x,t)|dx,$$

$$\text{where, } N(x) \geq 0,\ M(x) \geq 0, \qquad f > 0, f' > 0, f'' \geq 0,$$

$$\text{and } \lim_{u\to c-0} f(u) = \infty, \qquad \text{for some positive number } c.$$

Quenching for the above problem are widely studied recently. First we give hypoellipticity for the operator in the equation (1.1) in any interval containing $x = 0$, because for degenerate operator hypoellipticity is not easily derived. In fact at the origin the operaor changes types; parabolic to elliptic.

Second we prove differentiabilities of solutions of some types of functional equations. Specifically we investigate conditions by which all solutions $f(x)$