Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 209-222

CONNECTIONS BETWEEN TYPES OF SINGULARITIES IN DIFFERENTIAL EQUATIONS AND SMOOTHNESS OF SOLUTIONS FOR DIRICHLET BVPS

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Abstract. We present sufficient conditions for the existence of smooth sign-changing solutions of the singular Dirichlet boundary value problem with a positive parameter μ

$$(r(x)x')' = \mu q(t)f(t,x), \quad x(0) = x(T) = 0,$$

where f is singular at the point $x = 0$ of the phase variable x. By means of the study of connections between a type of the singularity and the smoothness of solutions we get conditions for non-existence of solutions of the above problem. Finally, we introduce a notion of w-solutions and give exact multiplicity results.
Keywords. Singular Dirichlet problem, smooth sign-changing solution, existence, strong and weak singularity, w-solution, multiplicity.
AMS subject classification: 34B16, 34B18.

1 Introduction

We will study the singular Dirichlet boundary value problem with a positive parameter μ

$$(r(x(t))x'(t))' = \mu q(t)f(t,x(t)), \quad t \in (0,T), \tag{1.1}$$

$$x(0) = x(T) = 0, \ \max\{x(t) : 0 \le t \le T\} \cdot \min\{x(t) : 0 \le t \le T\} < 0, \tag{1.2}$$

where $T \in (0,\infty)$ and f is singular at the point $x = 0$ of the phase variable x in the following sense

$$\lim_{x\to 0^-} f(t,x) = -\infty, \ \lim_{x\to 0^+} f(t,x) = \infty \quad \text{for } t \in [0,T]. \tag{1.3}$$

To give a finer classification of the singularity of f at $x = 0$ we first assume that f satisfies the condition

$$0 < f(t,x)\,\mathrm{sign}\, x \le g(x) \quad \text{for } (t,x) \in [0,T] \times D, \tag{1.4}$$