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SEMILINEAR ELLIPTIC SYSTEMS WITH LACK OF SYMMETRY

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Abstract. By means of a perturbative method introduced by Bolle we give a multiplicity result for a system of semilinear elliptic equations with non–homogeneous boundary conditions in the presence of a generic superquadratic odd nonlinear term.
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1 Introduction

Let $N \geqslant 1$ and $n \geqslant 2$. The main goal of this paper is to prove the existence of multiple solutions $u = (u_1, \dots, u_N) : \overline{\Omega} \to \mathbb{R}^N$ for the semilinear elliptic system

$$\begin{cases} -\sum\limits_{i,j=1}^{n}\sum\limits_{h=1}^{N} D_j(a_{ij}^{hk}(x)D_i u_h) = g_k(x,u) + \varphi_k(x) & \text{in } \Omega \\ u = \chi & \text{on } \partial\Omega \\ k = 1,..,N \end{cases} \qquad (\mathcal{S}_{\chi,\varphi,N})$$

where Ω is a smooth bounded domain in $\mathbb{R}^n$, $\varphi = (\varphi_1, \dots, \varphi_N) \in L^2(\Omega, \mathbb{R}^N)$, $\chi \in H^{1/2}(\partial\Omega, \mathbb{R}^N) \cap C(\partial\Omega, \mathbb{R}^N)$ and the coefficients $a_{ij}^{hk} \in C(\overline{\Omega}, \mathbb{R})$ are such that $a_{ij}^{hk} = a_{ji}^{kh}$. Assume that the Legendre–Hadamard condition holds, i.e., there exists $\nu > 0$ such that

$$\sum_{i,j=1}^{n}\sum_{h,k=1}^{N} a_{ij}^{hk}(x)\xi_i\xi_j\eta^h\eta^k \;\geqslant\; \nu\,|\xi|^2\,|\eta|^2 \qquad (1.1)$$

for all $x \in \Omega$ and $(\xi, \eta) \in \mathbb{R}^n \times \mathbb{R}^N$. Moreover, suppose that the nonlinear term $g = (g_1, \dots, g_N) \in C(\overline{\Omega} \times \mathbb{R}^N, \mathbb{R}^N)$ admits a potential G of class C^1 such that

$$\nabla_s G(x,s) = g(x,s), \quad G(x,0) = 0 \quad \text{for all } (x,s) \in \Omega \times \mathbb{R}^N$$