Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 169-179

A PARAMETER DEPENDENCE PROBLEM IN PARABOLIC PDES

Min He

Department of Mathematical Science
ent State University-Trumbull Campus
Warren, Ohio 44483 U.S.A.

Abstract

This paper is concerned with a family of densely defined operators with parameter dependent domains. A general result on differentiability with respect to parameters of C_0-semigroup on the entire space is presented. The result is used to discuss smoothness with respect to parameters of solutions of some parabolic PDEs.

AMS Subject Classification: 47D03, 35F10, 35F15, 35k05.

Key words: C_0-semigroup, differetiability, parameter, parabolic partial differential equation, Cauchy problem.

1 Introduction

Many PDE initial-boundary value problems can be formulated as the abstract Cauchy problem

$$\begin{aligned} u'(t) &= A(\varepsilon)u(t) + f(t, u(t), \varepsilon), \qquad \text{for } t \geq 0, \\ u(0) &= u_0, \end{aligned} \tag{1.1}$$

in which the domain of the operators $A(\varepsilon)$ is independent of parameter ε. However, when parameters enter into the boundary conditions, the domain of the operator varies as parameters change. Take for example a parabolic PDE initial-boundary value problem

$$\begin{aligned} & u_t = u_{xx}, && \text{for } t \geq 0, \\ & u(x,0) = f(x), && \text{for } x \in [0,1]\ , \\ & u(0,t) = \alpha u_x(0,t), && \\ & u(1,t) = -\beta u_x(1,t), && \alpha, \beta \geq 0. \end{aligned} \tag{1.2}$$

The associated abstract Cauchy problem is

$$\begin{aligned} \frac{du(t)}{dt} &= A(\varepsilon)u(t), \quad t \geq 0 \\ u(0) &= f \end{aligned} \tag{1.3}$$

on $X = (L^2[0,1], \|\cdot\|_{L^2})$, where

$$A(\varepsilon) = \tfrac{d^2}{dx^2}, \qquad \varepsilon = (\alpha, \beta) \in R^2_+ = \{(\alpha, \beta) \in R^2 \,\big|\, \alpha, \beta \geq 0\},$$

$$D(A(\varepsilon)) = \{\, u \in H^2[0,1] \,\big|\, u(0) - \alpha u'(0) = 0,\ u(1) + \beta u'(1) = 0 \,\}.$$