Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 157-167

MULTIPLE POSITIVE SOLUTIONS OF NONLINEAR BOUNDARY VALUE PROBLEMS

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Abstract. For any given positive integer N, we provide conditions on $w(x)$, $p(x)$, $f(y)$ which guarantee that the nonlinear boundary value problem $(1/w)(py')' + f(y) = 0$, $0 < x < 1$, $y(0) = y(1) = 0$ has at least N positive solutions. These results generalize previous work of J. Henderson and B. Thompson, as well as the authors.
Keywords. Nonlinear boundary value problems, multiple positive solutions, shooting.
AMS (MOS) subject classification: 34B15

1 Introduction

We consider nonlinear boundary value problems of the form

$$\frac{1}{w}(py')' + f(y) = 0, \quad 0 < x < 1, \tag{1}$$

$$y(0) = 0, \quad y(1) = 0, \tag{2}$$

where w, p are positive and continuous on $(0, 1)$, which allows singularities at the endpoints, and $f(y) \geq 0$ and continuous for $y \in \mathbb{R}$. For a given positive integer N, we provide conditions on the nonlinear function f which guarantee existence of N positive solutions. Our motivation comes from previous work in the case $w(x) \equiv p(x) \equiv 1$ of Henderson and Thompson [6], which uses a fixed point theorem of Leggett and Williams [8] and the relevant Green's functions to describe conditions on f which guarantee the existence of at least three positive solutions, and our previous paper [2] using initial value methods and extending those results to any number of positive solutions. Henderson and Thompson [6, 7] have also used their methods to prove results on the existence of three positive solutions to certain higher order boundary value problems, and Graef, et al [5] have approached the same problem using the Krasnosel'skii fixed point theorem; these papers have depended crucially on properties of Green's functions.

Consider the problem (1), (2) in the case that $w(x)$ and $p(x)$ are symmetric about $x = 1/2$. Clearly any solution of (1) on $[0, 1/2]$ with $y(0) = y'(1/2) = 0$ can be reflected across $x = 1/2$ to give a solution of (1), (2). Thus we shall focus on the boundary value problem