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POSITIVITY AND FINITE TIME EXTINCTION IN NONLINEAR DIFFUSION-ABSORPTION MODELS WITH CONVECTION

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Abstract. Previously, it has been shown that solutions of a so-called fast diffusion model, with appropriately weak absorption, are positive everywhere in the interior until extinction, at which time the solution is zero everywhere. If convection is added to the model, such behavior is likewise seen unless the convection is stronger than diffusion. In the case of dominant convection, the existence of traveling waves for the associated Cauchy problem gives rise to solutions which exhibit finite speed of propagation. The main roadblock to past investigations of such an effect of convection was the lack of a general subsolution/supersolution comparison theory. We discuss a new comparison theory for these models and then show how it may be used to obtain the above results. Herein, we restrict the majority of the discussion to a model in one space dimension and then give an indication of similar results for a higher dimensional model.
Keywords. nonlinear, diffusion, convection, absorption, extinction, Dirichlet problem

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1 Introduction

Solutions of the nonlinear diffusion-convection-absorption model

$$\begin{array}{ll} u_t = (u^m)_{xx} + \epsilon(u^n)_x - \lambda u^p & 0 < x < 1, t > 0 \\ u(0,t) = u(1,t) = 0 & t > 0 \\ u(x,0) = u_0(x) \geq 0 & 0 \leq x \leq 1 \\ m, n, p > 0; \lambda, \epsilon \geq 0; u_0 \epsilon L^\infty((0,1)) & \end{array} \qquad (DCA)$$

are expected to become extinct in finite time in the presence of a sufficiently strong absorption, i.e., $p < 1$. Similarly, analogous to known results for the porous medium equation $(m > 1, \epsilon = \lambda = 0)$, it is anticipated that solutions of (DCA) may vanish on a nonempty subset of $(0,1)$, even for small $t > 0$.

Li and Peletier established such results for the diffusion-absorption model $(\epsilon = 0)$. They showed that, in the so-called "fast diffusion" case $(0 < m < 1)$, if $p \geq m$, then there exists a time $T \equiv T(u_0) > 0$ such that $u > 0$ on $(0,1) \times (0,T)$ and $u \equiv 0$ on $[0,1] \times [T,\infty)$. If $0 < p < m < 1$, then they also showed that the second of these phenomena, "finite-time extinction," is still true. However, the first of these, "small-time positivity" is false for initial states with $u_0(x) \leq A \mid x - y \mid^{2/(m-p)}$ for some $y \in (0,1)$ and a positive constant A [12].