Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 103-115

POSITIVE SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

Lingju Kong and Qingkai Kong

Department of Mathematical Sciences
Northern Illinois University, DeKalb, IL 60115, USA

Abstract. For a class of boundary value problems consisting of a n-th order nonlinear functional differential equation and $(k, n-k)$ conjugate boundary conditions, we establish the existence of positive solutions. Existence conditions are obtained not only for one positive solution, but also for multiple and even countably infinitely many positive solutions. Our results provide extensions of some recent work in the literature on boundary value problems of ordinary differential equations.
Keywords. Positive solutions, existence, boundary value problems, deviating arguments, fixed point theorem on cones.
AMS (MOS) subject classification: 34K10

1 Introduction

Recently, Eloe and Henderson [6, 7] established the existence of positive solutions of the nonlinear $(k, n-k)$ conjugate boundary value problem (BVP) consisting of the equation

$$(-1)^{n-k}u^{(n)} = a(t)f(u), \quad 0 < t < 1, \tag{1.1}$$

and the boundary value conditions (BCs)

$$u^{(i)}(0) = 0, \quad 0 \le i \le k-1, \tag{1.2}$$

and

$$u^{(j)}(1) = 0, \quad 0 \le j \le n-k-1, \tag{1.3}$$

where $1 \le k \le n-1$. Their approach is based on the assumptions:

(A1) $f : [0,\infty) \to [0,\infty)$ is continuous;

(A2) $a : [0,1] \to [0,\infty)$ is continuous and does not vanish identically on any subinterval of $[0,1]$; and involve the limits

$$f_0 := \lim_{u\to 0+} \frac{f(u)}{u} \quad \text{and} \quad f_\infty = \lim_{u\to\infty} \frac{f(u)}{u}.$$