

## DOUBLE SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS WITH IMPULSE

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**Abstract.** A double fixed point theorem is applied to obtain the existence of at least two positive solutions for a right focal two point boundary value problem for a second order ordinary differential equation with impulse effect.

**Keywords.** Boundary value problem, impulse, double solutions.

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### 1 Introduction

Let  $0 < \tau < 1$  be fixed. We apply a recently established fixed point theorem to obtain multiple positive solutions of the nonlinear impulsive differential equation,

$$y'' + f(y) = 0, \quad t \in [0, 1] \setminus \{\tau\}, \quad (1)$$

subject to the underdetermined impulse condition,

$$\Delta y(\tau) = I(y(\tau)), \quad (2)$$

and satisfying the right focal boundary conditions,

$$y(0) = y'(1) = 0, \quad (3)$$

where  $\Delta y(\tau) = y(\tau^+) - y(\tau^-)$ ,  $f : \mathbb{R} \rightarrow [0, \infty)$  is continuous, and  $I : [0, \infty) \rightarrow [0, \infty)$  is continuous. By a positive solution, we shall mean *positive with respect to a suitable cone*.

Fixed point theorems have long been used in establishing solutions to nonlinear problems. Zeidler's book [29] documents a good part of this. Included in these uses of fixed point theorems are their applications to obtain