Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **9 (2002)** 489-505

GLOBAL SUPERCONVERGENCE ANALYSIS IN $W^{1,\infty}$-NORM FOR GALERKIN FINITE ELEMENT METHODS OF INTEGRO-DIFFERENTIAL AND RELATED EQUATIONS

Tang Liu[1], Liping Liu[2], Ming Rao[3] and Shuhua Zhang[1]

[1]Department of Mathematics
Tianjin University of Finance and Economics, Tianjin 300222
LiuHui Center for Applied Mathematics
Nankai University 300071, Tianjin University 300072
[2]Department of Mathematical Sciences
Lakehead University, Thunder Bay, Ontario P7B 5E1
[3] Department of Chemical and Materials Engineering
University of Alberta, Edmonton, Alberta T6G 2G6

Abstract. The object of this paper is to investigate the global superconvergence of finite element approximations in $W^{1,\infty}$-norm to solutions of parabolic and hyperbolic integro-differential equations, and also of equations of Sobolev and viscoelasticity type. Behind the analysis the estimates for the regularized Green's functions with memory terms, the concept of Ritz-Volterra projection and the interpolation postprocessing technique will be seen to play important roles. As by-products, the global superconvergence can also provide efficient a posteriori error estimators.

Keywords. Partial integro-differential equations, Galerkin finite element methods, Ritz-Volterra projection, regularized Green's functions, interpolation postprocessing, global superconvergence, a posteriori error estimators.
AMS (MOS) subject classification: 76S05, 45K05, 65M12, 65M60, 65R20.

1 Introduction

Our first purpose in this paper is to study the global superconvergence in maximum norm for the derivatives of the time-continuous Galerkin finite element solutions of parabolic integro-differential equation of the form

$$\begin{aligned} &u_t + A(t)u + \int_0^t B(t,s)u(s)ds = f(t) \ \text{ in } \ \Omega \times J, \\ &u = 0 \ \text{ on } \ \partial\Omega \times J, \\ &u(0) = u_0(x) \ \ x \in \Omega, \end{aligned} \tag{1.1}$$

where $\Omega \subset R^2$ is an open bounded domain with smooth boundary $\partial\Omega$, $J = (0,T)$ with $T > 0$. Here, $A(t)$ is a self-adjoint positive definite linear elliptic partial differential operator of second order, and $B(t,s)$ an arbitrary second-order linear partial differential operator, both with coefficients depending