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SINGULAR INTEGRAL EQUATIONS ARISING IN HOMANN FLOW

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Abstract. Positive solutions are established for the singular integral equation $y(t) = \int_0^1 k(t,s)\,[g(y(s)) + h(y(s))]\,ds,\ t \in [0,1]$. Our nonlinearity may be singular at $y = 0$ and our theory includes a problem which arises in the boundary layer theory in fluid mechanics.

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1. Introduction.

In the axisymmetric stagnation flow (i.e. Homann flow) the Navier–Stokes equation can be reduced to the third order Falkner–Skan equation (for $f(\eta)$)

$$f''' + f\,f'' + \frac{1}{2}\,(1 - (f')^2) = 0, \quad 0 < \eta < \infty \tag{1.1}$$

with boundary conditions

$$f(0) = 0,\ f'(0) = 0 \ \text{ and } \ f'(\infty) = 1. \tag{1.2}$$

Assume $f(\eta)$ is a solution of (1.1), (1.2) and $f''(\eta) > 0$ for all $\eta \ge 0$. Then $\eta = g(t)$, the inverse function to $t = f'(\eta)$, exists and is strictly increasing on $(0,1)$ with $g(0) = 0$ and

$$t = f'(g(t)) \ \text{ for all } \ t \in (0,1).$$

Differentiate with respect to t to obtain

$$w(t) \equiv f''(g(t)) = \frac{1}{g'(t)},\ \ 0 < t < 1.$$

For simplicity a prime will denote differentiation with respect to t or η. Substitute $\eta = g(t)$ into (1.1), and use $w'(t) = f'''(g(t))\,g'(t) = \frac{f'''(g(t))}{w(t)}$, to obtain

$$w'(t)\,w(t) + f(g(t))\,w(t) + \frac{1}{2}\,(1 - t^2) = 0,\ \ 0 < t < 1. \tag{1.3}$$

Divide by w and differentiate with respect to t to obtain

$$w''(t) = \frac{t\,w(t) + \frac{1}{2}\,(1-t^2)\,w'(t)}{w^2(t)} - \frac{t}{w(t)} = \frac{(1-t^2)\,w'(t)}{2\,w^2(t)},\ \ 0 < t < 1.$$